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Laminar natural convection heat transfer from inclined surfaces

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INTRODUCTION

NATURAL convection heat transfer from flat plates is a problem which is of interest in a variety of industrial applications. As a result, these flows have been the subject of numerous theoretical and experimental studies. In addition to the limiting cases of flow adjacent to vertical and horizontal surfaces, the intermediate case of inclined plate heat transfer has also been examined by a number of investigators.

Theoretical predictions of laminar heat transfer correlations can be obtained from similarity solutions for vertical plates as described by Sparrow and Gregg [1] and Yang [2], who followed the classic work by Ostrach [3]. Extension of similarity solutions to inclined surfaces has been discussed by Chen and Yuh [4]. Similarity solutions for a constant flux surface predict a correlation of the form

$$Nu_x = C(Gr_x^*)^{1/5} \quad (1)$$

where the local Nusselt number and the modified local Grashof number are defined by

$$Nu_x = \frac{h_x x}{k} \quad \text{and} \quad Gr_x^* = Gr_x\, Nu_x = \frac{g\beta q_w x^4}{k\nu^2}. \quad (2)$$

A theoretical prediction for heat transfer to room temperature water can be obtained from the similarity solution for a Prandtl number of 6.14 (corresponding to water at 25°C). In terms of Ra_x^*, the modified Rayleigh number, the predicted relation can be shown [5] to be

$$Nu_x = 0.587(Ra_x^*)^{1/5}. \quad (3)$$

Experimental studies of heat transfer adjacent to inclined surfaces began with work conducted with isothermal surfaces. Rich [6] was the first to show that heat transfer coefficients could be correlated for inclined surfaces by using only the component of gravity parallel to the surface in the Grashof number ($g \cos\theta$ instead of g where θ is the angle from vertical). Rich's result, which was obtained for an isothermal surface inclined at up to 40° from vertical has been supported by numerous subsequent investigators working with both isothermal surfaces (e.g. refs. [7, 8]) and with uniform heat flux surfaces (e.g. refs. [9, 10]).

Available correlations for uniform heat flux inclined surfaces stem primarily from the studies by Vliet [9]. Vliet arrived at a correlation of

$$Nu_x = 0.60(Gr_x^* \cos\theta\, Pr)^{0.20} \quad (4)$$

using a heated foil and both water and air as working fluids. Vliet made measurements of temperature differences as small as 3°F (1.7°C) and estimated his measurement error for the Nusselt number at 5% for high heat fluxes and 15% for low heat fluxes. The lowest Rayleigh numbers measured were in the neighborhood of $Ra_x^* = 6\times10^6$. The highest heat flux measurements extended to $Ra_x^* = 2\times10^{15}$; well into the turbulent flow region. Inclinations measured ranged from vertical to 60° from vertical. Another set of data for inclined surfaces was obtained using a heated foil in air [10], and resulted in a correlation of

$$Nu_x = 0.55(Gr_x^* \cos\theta\, Pr)^{0.20}. \quad (5)$$

In this instance, measurements for upward facing plates were made only for 30° from vertical and for vertical. The laminar data extended down to modified Rayleigh numbers as low as 5×10^8. Since the experiments were conducted in air, corrections were required for radiative and conductive losses of heat. These corrections amounted to 18–23% of the heat dissipated. Deviation of the Nusselt numbers from the recommended correlation appears to be in the range of 10–20% for the upward facing inclined data.

Here, we describe measurements of laminar inclined surface heat transfer data which were obtained over a wider range of experimental conditions and with more precision than has been previously reported. The experimental scatter of Nusselt numbers measured by the techniques presented in this investigation was typically less than 1–2%. The results described are an offshoot of a set of studies which were directed toward an improved understanding of atmospheric mountain slope flows [5].

With measurements of greater precision, the accuracy and range of applicability of the similarity solutions can be more adequately assessed. In particular, the validity of applying the vertical plate correlations to near-horizontal inclinations by a $\cos\theta\, Ra_x^*$ correction is addressed. In addition, the low Rayleigh number behavior of the heat transfer coefficient is described. Isothermal data reported in ref. [11] indicate that there is a lower limit to the linear region of heat transfer correlations associated with a change of regime from convection heat transfer to conduction. Such measurements for uniform flux surfaces require a high degree of precision in determining the surface to bulk temperature difference. As Holman [11] pointed out, experimental scatter of ±20% is typical for these types of experiments which has made it difficult to address these problems in previous investigations.

EXPERIMENTAL DESIGN

In this section we begin by briefly describing the equipment used for these experiments. Drawings and more extensive details of the construction are available elsewhere [5] and are not included here. The heated surface used for these experiments was 0.3048 m long and 0.1524 m wide and consisted of a 0.0254 mm stainless steel foil backed by 0.0191 m thick extruded polystyrene insulation and a 0.0127 m acrylic plate. The heated foil (made up from stainless steel #304

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NOMENCLATURE

C	coefficient appearing as the leading term of the correlating equation
g	acceleration of gravity
Gr_x	local Grashof number, $g\beta(T_w - T_\infty)x^3/\nu^2$
Gr_x^*	modified local Grashof number ($= Gr_x\ Nu_x$), $g\beta q_w x^4/(k\nu^2)$
h_x	local heat transfer coefficient, $q_w/(T_w - T_\infty)$
k	thermal conductivity
m	coefficient appearing as an exponent of the correlating equation
Nu_x	local Nusselt number, $h_x x/k$
Pr	Prandtl number, ν/α
q_w	wall heat flux
Ra_x^*	modified local Rayleigh number, $Gr_x^*\ Pr$
T_∞	bulk temperature
T_f	film temperature, $(T_w + T_\infty)/2$
T_w	wall temperature
ΔT	temperature difference, $T_w - T_\infty$
x	distance from the leading edge of the heated surface.

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
θ	inclination from vertical [deg]
ν	kinematic viscosity.

shim stock) was attached to the polystyrene using double sided rubber tape. Acrylic side walls of height 0.30 m were attached to prevent inflow from the sides. Experiments were conducted in a 61 cm deep Lexan® tank (dimensions $61 \times 61 \times 122$ cm) which was filled with water. The leading edge of the heated surface was situated about 0.05 m above the bottom of the tank and the trailing edge of the heated surface was always at least 0.20 m below the top of the water. Power was supplied by a Rapid Electric d.c. power supply (0–30 V, 0–200 A). Connections to the foil were made using stainless steel knife edge terminals between which the foil was tightly stretched. Six temperature sensors were imbedded between the tape and the heated foil for surface temperature measurements. The sensors were evenly spaced along the centerline of the heated surface at distances from 0.025 to 0.275 m from the leading edge.

As a preliminary to the experiments described in this paper, trial measurements were made with a variety of temperature sensors. Thermistor type sensors were eventually chosen since they yield highly accurate signals with a large temperature coefficient which permits great precision of measurement. The sensors used (Thermometrics, Edison, New Jersey; Cat. #AB6C8-BR16KA103J) had a nominal diameter of 0.406 mm and were covered with a conformal epoxy coating for immersion in conductive fluids. The sensors had a characteristic response time of ~0.08 s (in water) and were used for both bulk fluid and surface temperature measurements. All sensors were calibrated to an accuracy of ±0.1°C against larger diameter thermistors which were manufactured to this standard (Yellow Springs Instruments, Yellow Springs, Ohio; Cat. #44131). Data sampling was accomplished using a microcomputer equipped with an A/D board (Tecmar, Cleveland, Ohio; Model #AD-212). Further details of the calibration procedure and of the data sampling arrangements are described elsewhere [5].

A critical factor in these experiments was that the sensors have very good thermal contact with the surface. The temperature sensors were set into shallow grooves which were melted into the tape which held the heated foil to the styrofoam backing. The grooves were filled with thermally conductive paste (Omega Inc., Stamford, Connecticut; Cat. #OT-201) which wet the sensors and the foil and gave the best possible thermal contact. To further improve the thermal contact, an acrylic back plate was constructed with threaded pistons in the areas of the sensors. These were carefully raised upward to press the sensor plugs firmly against the foil.

Before beginning an experiment, the water was deaerated, filtered of particles down to 0.5 μm and thoroughly mixed to ensure a uniform bulk temperature. After waiting ~30 min (sufficient for all visible fluid motion in the tank to cease) a vertical traverse of the bulk fluid was made using a fluid temperature sensor in order to confirm that there was no residual temperature gradient in the bulk fluid. The temperature sensor was then placed near the trailing edge of the plate in order to detect any heating of the bulk fluid.

An experiment was initiated by turning on the power supply to the plate and waiting 3–5 min to allow the surface temperature to stabilize to the new value. The surface temperature was then sampled for 5–10 min to obtain an average value for each location along the plate over this time period. After determining the surface temperatures for a given heat flux, the voltage across the plate was increased and the measurement procedure repeated for the new heat flux. A heated region in the bulk fluid at the top of the tank developed as an experiment proceeded but it was found that 4–7 flux levels could be measured in this manner before the heated bulk fluid penetrated down to the depth of the plate. During the experiments described here, the heat flux was varied over a range from ~6 to ~60000 W m^{-2}. The corresponding temperature differences ranged from ~0.05 to ~80°C.

To determine the temperature differences between the surface and the bulk fluid along the plate for a given heat flux it is normally sufficient to subtract the measured bulk temperature from the measured value of the surface temperature. In the present case, to obtain accurate measurements for the lowest Rayleigh number range, it was necessary to measure temperature differences as low as 0.05°C. To achieve the degree of accuracy desired, it was necessary to determine temperature differences by an alternative method, which was to take the temperature rise to an individual sensor relative to its initial unheated state. Since the precision and repeatability of an individual sensor was better than 0.01°C this method afforded a much more accurate determination for very small temperature differences. A small correction (usually ~0.01–0.02°C) was applied to the measured temperature rise to correct for the slow variation of bulk temperature due to unsteady room temperature conditions as measured by the bulk fluid sensors. Smoothing of the bulk temperature data was accomplished by fitting it to a linear trend of variation with time. This ensured that the accuracy of the smallest temperature differences was not adversely affected by any random scatter (±0.005°C) of individual bulk temperature measurements. With these techniques, the error in measurement of the temperature difference was estimated to be $\sim 0.01\Delta T + 0.005$°C.

The current supplied by the power supply was determined by measuring the voltage drop across a current shunt resistor. This gave the current through the foil to an accuracy of ±0.5%. The resistance of the foil and the temperature coefficient of resistivity were taken from separate measurements made with the shim stock, permitting the heat flux to be calculated using the electrical determination of power dissipated, along with the known area of the resulting heated surface. Based on the errors in current and resistance measurement, the heat flux thereby determined was estimated to have a likely error of ±1%.

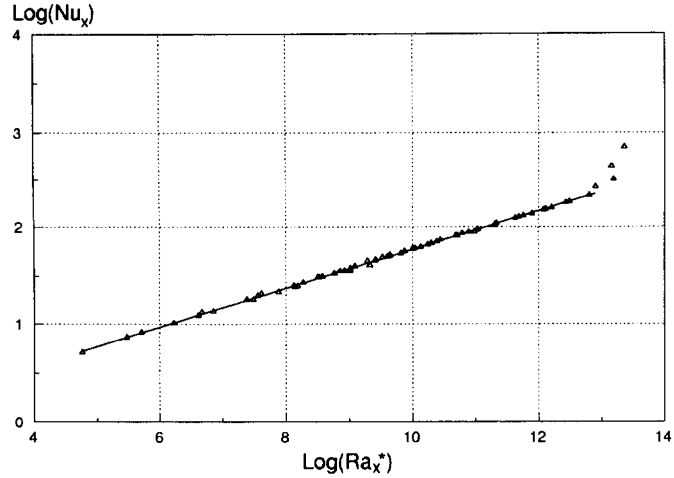

FIG. 1. Laminar local heat transfer data—vertical surface. Vertical uniform flux surface in a uniform temperature bulk fluid. Plot of local Nusselt number vs modified local Rayleigh number. Solid line: $Nu_x = 0.587(Ra_x^*)^{1/5}$.

Heat transfer correlations were obtained by performing best fit calculations using a general least squares procedure (LFIT) extracted from ref. [12]. This procedure incorporates weighting factors which were individually evaluated for each data point. The weighting was based on the uncertainty of each data point so that data at lower Nusselt and Rayleigh numbers weighed less heavily than data at higher values. For the experimentally obtained Nusselt numbers the resulting error estimates averaged 2.5% with the largest estimated errors being $\sim 10\%$ for the lowest Nusselt number data.

EXPERIMENTAL RESULTS

Vertical surface

Figure 1 shows the data which were collected for measurements with a vertical plate. The local Nusselt number as determined experimentally for various heat fluxes is plotted as a function of the modified local Rayleigh number, Ra_x^*. Physical properties of water were evaluated at the local film temperature, T_f, where

$$T_f = (T_w + T_\infty)/2. \quad (6)$$

Relations which were used to obtain the physical properties of water as a function of temperature are detailed in ref. [5]. The best fit correlation obtained using the weighted fit described by ref. [12] was, for the vertical case

$$Nu_x = 0.585(Ra_x^*)^{0.200}. \quad (7)$$

Only the data for $\log(Ra_x^*) < 12.8$ were included in the fitting calculation, so as to exclude any turbulent or transition data. The data extend as low as $Ra_x^* = 5\times 10^4$ and compare favorably with the data of Vliet [9] which reach down to $Ra_x^* = 6\times 10^6$ and Qureshi and Gebhart [13] which reach down to $Ra_x^* = 2\times 10^6$. The data also match up very well with the predicted correlation which would be expected from similarity solutions to the boundary layer equations. This line is shown on Fig. 1 for comparison to the experimental data. The scatter of these data was typically less than 1–2% from the best fit line.

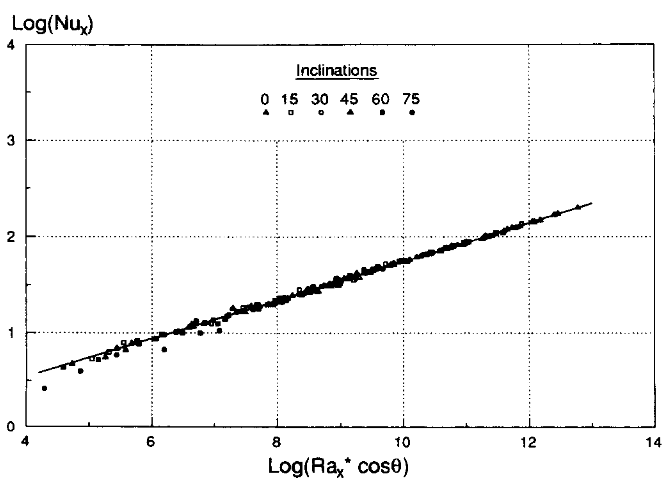

FIG. 2. Laminar local heat transfer data—angles 0°, 15°, 30°, 45°, 60°, 75°. Uniform flux surface in a uniform temperature bulk fluid at various inclinations from vertical. Plot of local Nusselt number vs modified local Rayleigh number. Solid line: $Nu_x = 0.587(Ra_x^* \cos\theta)^{1/5}$.

Table 1. Weighted best fit coefficients for correlating equation: $Nu_x = C(Ra_x^* \cos\theta)^m$

Inclination, θ	Laminar range, $\log(Ra_x^* \cos\theta)$	Two parameter fit C	m	One parameter fit; $m = 1/5$ C	m
0	4.7–12.8	0.585	0.200	0.584	1/5
15	5.0–12.0	0.592	0.199	0.586	1/5
30	5.7–10.0	0.573	0.201	0.584	1/5
45	5.2–9.0	0.615	0.195	0.576	1/5
60	4.5–8.0	0.524	0.205	0.568	1/5
75	4.2–7.1	0.237	0.259	0.553	1/5

Inclined surfaces

The laminar portion of the data obtained for inclined surfaces is presented on a single plot in Fig. 2. Data are shown for six inclinations which ranged from 0° to 75° from vertical. Precise measurements focusing on transition behavior were not a goal of this work. However, the observed ranges of the transition regions agreed well with the observations of Vliet [9]. The $\cos\theta$ correction was applied to the Rayleigh number, and as can be seen, all of the data collected are well represented by a single correlating line. Some deviations become evident in the data for $\theta = 75°$. However, the data points for $\theta = 75°$ are found mostly in the low Nusselt number range due to the earlier transition to turbulence as the inclination begins to approach horizontal. As a result, these data are subject to markedly greater experimental error and should be regarded with some caution.

Table 1 shows the calculated best fit coefficients for all inclinations, including vertical. As can be seen, the two parameter best fit line at $\theta = 75°$ had an exponent of 0.259 which indicates a local heat transfer coefficient which is nearly independent of distance along the surface (as has been found elsewhere for turbulent flow [7–10]). The two parameter fit was very sensitive to the value of m. Thus, small variations in the exponent could make the constant, C, appear to be varying more than the actual variation of the heat transfer coefficient. Accordingly, further calculations were made to obtain the best fit value for C with a forced exponent of 1/5 as suggested by the boundary layer theories for a uniform flux surface. With the data presented in this form it becomes evident that the data can be very well represented by the theoretically obtained coefficients $C = 0.587$ and $m = 1/5$. The precision of the data is such that a small but significant trend is evident with θ that is not completely eliminated by the $\cos\theta$ correction. As θ increases, $Nu_x = 0.587(Ra_x^* \cos\theta)^{1/5}$ tends to overestimate the Nusselt number slightly. For $\theta = 75°$ the average deviation would be about 6%, still reasonable accuracy considering the greater experimental errors incurred in measuring these data points.

SUMMARY

Utilizing thermistor type temperature sensors, local heat transfer coefficients have been determined for laminar natural convection from uniform flux heated surfaces at a variety of inclinations. The sensors used permitted temperature differences as small as 0.05°C to be accurately determined. The resulting heat transfer data agreed very well with the correlation predicted by similarity solutions for vertical plates and covered a range which extended two orders of magnitude lower in Ra_x^* than previously available results. For inclined surfaces, a $\cos\theta$ correction to the gravitational acceleration was used to adjust the Rayleigh number values. The heat transfer was very well correlated by the similarity solution modified to account for the direction of gravitational acceleration

$$Nu_x = 0.587(Ra_x^* \cos\theta)^{1/5}. \quad (8)$$

This relationship resulted in only a slight overestimate for the heat transfer as inclination increased away from vertical.

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